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Higher derivative operators as loop counterterms in one-dimensional field theory orbifolds

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Abstract: Using a 5D $N = 1$ supersymmetric toy-model compactified on $S_1/(Z_2 \times Z_2')$, with a "brane-localised" superpotential, it is shown that higher (dimension) derivative operators are generated as one-loop counterterms to the (mass)^2 of the zero-mode scalar field, to ensure the quantum consistency of the model. Such operators are just a result of the compactification and integration of the bulk modes. They are relevant for the UV momentum scale dependence of the (mass)^2 of the zero-mode scalar field, regarded as a Higgs field in more realistic models. While suppressed for a small compactification radius R, these operators can affect the predictive power of models with a large value for R. A general method is also provided for a careful evaluation of infinite sums of 4D divergent loop-integrals (of Feynman diagrams) present in field theory orbifolds. With minimal changes, this method can be applied to specific orbifold models for a simple evaluation of their radiative corrections and the overall divergences.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Supersymmetry Phenomenology, Supersymmetric Effective Theories.

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1. Introduction

The physics of extra dimensions received a renewed interest in the context of field theory orbifolds which attempt to derive SM-like models (or supersymmetric versions) from compactifications of a higher dimensional theory (see for example refs. [1]–[11]). In this work we consider a rather simple 5D model compactified on $S_1/(Z_2 \times Z_2')$ to illustrate further quantum effects, so far neglected.

The motivations of this study are described in the following. In the context of an orbifold compactification of a 5D model, a natural question to ask is whether higher dimension operators are important at one loop (assuming they were not introduced in the tree level action). The study of higher dimensional operators is important for they can affect the predictive power of the model by introducing additional parameters in the theory. This may be particularly relevant in models with a low compactification scale. For renormalisable theories, if they are not added at the tree level, they will not be generated as counterterms by radiative corrections. However, in higher dimensional (non-renormalisable) models such as that considered here this is not true. In some cases higher dimensional (derivative) operators are radiatively generated as one-loop counterterms to the couplings or masses of the model, see for example the case of gauge couplings [12, 13] in two-dimensional compactifications. While their presence there was related to the two-dimensional nature of the compactification, we shall see that the same technical reason which generated them at one-loop[1] is at work even in the case of simpler, one-dimensional orbifolds.

A second motivation is to provide in an orbifold model an wilsonian picture for the dependence on the high (UV) scale of the radiative corrections (induced by the Kaluza-

[1] This was due to the levels degeneracy and the presence of two Kaluza-Klein sums acting on the loop integral [13].

Klein (KK) modes) to the couplings or masses of the theory. This is done in a way which keeps manifest symmetries generic in this class of models (gauge, etc) (for a discussion of symmetries, anomalies and regularisation issues in field theory orbifolds see [14]–[18]). The technique we employ can be applied whenever one must sum infinitely many KK contributions from 4D divergent Feynman diagrams and only requires a DR regulator ϵ ($\epsilon \to 0$) for the associated loop-integrals. To find the scale dependence of the radiative corrections in the DR scheme one must evaluate the relevant loop diagrams for non-vanishing external momentum (q^2). Usually one takes $q^2 = 0$ since the external momentum is much smaller than the mass of the KK modes in the loop. In general this is appropriate for a high compactification scale, but in this work we keep $1/R$ arbitrary. Moreover, at $q^2=0$ additional would-be divergences of radiative corrections (such as for example $q^{2n}R^{2n-2}/\epsilon$ for a one-loop corrected (mass)^2), induced by multiple sums over the KK modes, cannot be "seen".[2] With these remarks, the momentum scale q^2 is regarded as the physical scale of the wilsonian picture of the theory.

As an example, we shall consider a brane-localised superpotential in a minimal 5D $N=1$ supersymmetric model compactified on $S^1/(Z_2 \times Z_2')$ and consider the one-loop effect on the (mass)^2 of a KK zero-mode scalar field (hereafter denoted $\phi_{H,0}$). We thus address the dependence of this (mass)^2 on the momentum scale q^2 to gain information on the UV regime of the model. Our findings are relevant for field theory orbifold models with similar compactifications [1], [4]–[7], where our zero-mode scalar field $\phi_{H,0}$ is usually identified with a Higgs field. In such models $m^2_{\phi_{H,0}}(0) \sim 1/R^2$, with implications for phenomenology [1]–[11]. Here we compute $m^2_{\phi_{H,0}}(q^2)$ and its dependence under the (UV) scaling of q^2. Such dependence cannot be supplied by string or field theory calculations of the loop expansion of the corresponding scalar potential (derived at $q^2=0$ of the external legs) with tree level (fixed) values of the couplings.

Another motivation is to clarify the *exact* link between higher derivative operators (HDO) on orbifolds and the 5D nature of the initial theory. As we shall see shortly, it turns out that the presence of HDO is strictly related to the *infinite* Kaluza-Klein sums associated with the initial 5D theory. Therefore, the existence of HDO is just an effect of compactification of a higher dimensional theory which - although supersymmetric - is nevertheless non-renormalisable, and this is an interesting finding. It is useful to note the connection of such aspects of compactification to dimensional crossover, coarse graining and non-perturbative effects, largely studied in the context of Ising-like systems [19, 20, 21]. Such a point of view can provide a rich insight into an orbifold compactification, the dependence of the couplings or masses on q^2 and the decoupling of *infinitely* many states, while varying q^2 relative to $1/R^2$.

Finally we outline a generalisation of the method used in the example below for computing radiative corrections in field theory orbifolds. The method carefully evaluates the infinite sums of divergent loop integrals of the corrections, applies for a very general Kaluza-Klein mass spectrum and is presented in appendix C. We hope this will help the careful study of more realistic models.

[2]Such divergences can be due to higher derivative operators, see discussion below.

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2. Higher derivative operators as one-loop counterterms.

To illustrate these ideas we consider the one-loop corrected (mass)^2 of a (zero-mode) scalar field, in a 5D $N = 1$ supersymmetric toy-model with hypermultiplet fields compactified on $S_1/(Z_2 \times Z_2')$. Here S_1 has radius R and we have the identification $x_5 \to x_5 + 2\pi R$. Under $Z_2 : x_5 \to -x_5$, while under $Z_2' : x_5 \to -x_5 + \pi R$. An $N = 2$ hypermultiplet decomposes into two four-dimensional $N = 1$ chiral superfields M and M^c of components $M = (\phi_M, \psi_M)$ and $M^c = (\phi_M^c, \psi_M^c)$, with opposite quantum numbers. We also assume the index M runs over the superfields Q, U, D, L, E. Their parities are fixed such as their (component) fermion fields are massless zero-modes fields, as one would like in a realistic model aimed at reproducing the low-energy SM "massless" fermionic spectrum. This is made possible with the parities assignment in the table below. The analysis can be extended to three families of fields, as any realistic model would require. Further, an extra hypermultiplet is introduced, of $N = 1$ components $H = (\phi_H, \psi_H)$ and $H^c = (\phi_H^c, \psi_H^c)$ whose Z_2' parity is chosen opposite to that of the fields M. Such (allowed) choice ensures that the massless zero-mode is now a scalar rather than a fermion. In a more detailed model [5] (see also [7]) this scalar may be identified with the usual Higgs boson of the SM. This different parity choice under Z_2' distinguishes between the two types of hypermultiplets on $S_1/(Z_2 \times Z_2')$. The (Z_2, Z_2') parities and the KK masses are then

4D fields	$\phi_{M,n}$	$\phi^c_{M,n}$	$\psi_{M,n}$	$\psi^c_{M,n}$	$\phi_{H,n}$	$\phi^c_{H,n}$	$\psi_{H,n}$	$\psi^c_{H,n}$
(Z_2, Z_2')	$+,-$	$-,+$	$+,+$	$-,-$	$+,+$	$-,-$	$+,-$	$-,+$
modes n	≥ 0	≥ 0	≥ 0	≥ 1	≥ 0	≥ 1	≥ 0	≥ 0
(mass)$\times R$	$2n{+}1$	$2n{+}1$	$2n$	$2n$	$2n$	$2n$	$2n{+}1$	$2n{+}1$

The KK fields come with the wavefunctions

$$\begin{aligned}
&\phi_{M,n} : \cos\frac{(2n+1)x_5}{R}, \quad \phi^c_{M,n} : \sin\frac{(2n+1)x_5}{R}, \quad \psi_{M,n} : \cos\frac{2nx_5}{R}, \quad \psi^c_{M,n} : \sin\frac{2nx_5}{R} \\
&\psi_{H,n} : \cos\frac{(2n+1)x_5}{R}, \quad \psi^c_{H,n} : \sin\frac{(2n+1)x_5}{R}, \quad \phi_{H,n} : \cos\frac{2nx_5}{R}, \quad \phi^c_{H,n} : \sin\frac{2nx_5}{R}
\end{aligned} \tag{2.1}$$

The next step would be to introduce vector multiplets and the corresponding gauge interactions.[3] However, we will not do so, and restrict the spectrum to the matter content given above. Although this assumption is not phenomenologically viable, it is appropriate to illustrate the main point of this work. Further, the interaction that we consider is

$$L = \int dx_5 \, \frac{1}{2} \left[\delta(x_5) + \delta(x_5 - \pi R)\right] \int d^2\theta \; \lambda \; Q \, U \, H + \text{h.c.} \tag{2.2}$$

Note that *all fields* Q, U, H in (2.2) are bulk fields (have associated KK modes). Here $\lambda \sim (\text{mass})^{-3/2}$ and the fields have dimension (mass)$^{3/2}$. This minimal interaction is very common in 5D extensions of the SM (see for example [5, 7]), with important implications

[3] If included, with this hypermultiplet content, a quadratic divergence exists due to Fayet-Iliopoulos term [22].

for phenomenology,[4] and this motivates its further analysis. One may include additional Yukawa interactions at the other fixed points of the orbifold ($\pm\pi R/2$). Such interactions can involve fields Q, D, H or L, E, H (with $\phi_M \to \phi_M^{c\dagger}$). A detailed analysis should also consider overlapping effects of the two types of fixed points and associated interactions, since only then would the "global" structure of $S_1/(Z_2 \times Z_2')$ be seen. This involves however higher loop calculations and is beyond the purpose of this paper. For simplicity the discussion below is restricted to the interaction given in (2.2).

Our purpose is to compute the dependence of the one-loop correction to the (mass)^2 of the zero-mode scalar $\phi_{H,0}$ of H, on the (euclidean) external momentum q^2 in the corresponding Feynman diagrams induced by interaction (2.2). Previous calculations of one-loop corrected (mass)^2 in 5D orbifold models were restricted to $q^2 = 0$ when they simplify considerably. Further, it is usually thought that, to investigate the leading UV behaviour of the scalar field mass in 5D orbifolds, it is sufficient to consider $m_{\phi_{H,0}}(0)$. However, in general in a *non-renormalisable* model with infinitely many states in the loop, this picture is incomplete and new effects emerge, as we shall see shortly.

After expanding interaction (2.2) in component fields[5] and Fourier (Kaluza-Klein) modes corresponding to x_5, one has three one-loop Feynman diagrams for the two-point Green function with the zero-mode scalar $\phi_{H,0}$ as external legs. The first has two vertices, with both $\phi^c_{Q,m}$ and $\phi_{U,k}$ in the loop, coupling together to $\phi_{H,0}$ (and a similar contribution with $Q \leftrightarrow U$). In addition to this there is a one-vertex diagram with only $\phi_{Q,k}$ in the loop, which couples to $\phi_{H,0}$ (a similar contribution with $\phi_{U,k}$). For the fermionic contribution there is a diagram with both $\psi_{U,k}$ and $\psi_{Q,m}$ in the loop, coupling together to $\phi_{H,0}$. After evaluating these one-loop contributions in the DR scheme, one finds (up to an overall colour factor, not written explicitly) the following formulae for the bosonic ($\mathcal{B}$) and fermionic ($\mathcal{F}$) effects to the mass $m^2_{\phi_{H,0}}$ of the zero-mode scalar $\phi_{H,0}$

$$
\begin{aligned}
-\, i m^2_{\phi_{H,0}}(q^2)\Big|_{\mathcal{B}} &= -i f_t^2 \sum_{k,m\geq 0} \eta^2_{k,\phi}\eta^2_{m,F}\,\mu^{4-d} \int \frac{d^d p}{(2\pi)^d} \frac{(p+q)^2}{(p^2 + m^2_{\phi_{U,k}})((p+q)^2 + m^2_{\phi^c_{Q,m}})} \\
-i\, m^2_{\phi_{H,0}}(q^2)\Big|_{\mathcal{F}} &= i f_t^2 \sum_{k,m\geq 0} \eta^2_{k,\psi}\eta^2_{m,\psi}\mu^{4-d} \int \frac{d^d p}{(2\pi)^d} \frac{p.(p+q)}{(p^2 + m^2_{\psi_{U,k}})((p+q)^2 + m^2_{\psi_{Q,m}})}
\end{aligned}
\tag{2.3}
$$

Here $d = 4 - \epsilon$, with $\epsilon \to 0$ the DR regulator of the momentum integrals, and μ is the usual finite, non-zero mass scale that dimension regularisation introduces. f_t is the 4D coupling, related to λ by $\lambda = (\pi R)^{3/2} f_t$. Further, $\eta^2_{k,\phi} = \eta^2_{m,F} = 1$ for the bosonic sector and $\eta^2_{0,\psi} = 1/2$, $\eta^2_{k,\psi} = 1$, $(k \geq 1)$ for fermions. These are wavefunction coefficients which take account of different normalisation of the zero-mode fields. From the table of parities one has

$$
m_{\phi^c_Q,k} = m_{\phi_U,k} = \frac{2}{R}\left[k + \frac{1}{2}\right], \quad k \geq 0; \qquad \text{and:} \qquad m_{\psi_U,k} = m_{\psi_Q,k} = \frac{2}{R}k, \quad k \geq 0. \tag{2.4}
$$

[4]This minimal model may also be recovered in the formal limit of vanishing gauge couplings of the model in [5].

[5]See reference [23]. For interaction (2.2) in full component form (onshell) see for example [5], also [7].

This mass spectrum is special in that it allows one to re-write the one-loop correction in (2.3) as sums over the whole set $\mathbb{Z}$ of integers[6] and this simplifies our calculation considerably. Appendix C extends the calculation of (2.3) to a more general KK mass spectrum than that in (2.4), (which takes account of potential mass shifts induced by levels mixing if present), and with important changes for the overall divergences of $m_{\phi_{H,0}}$. To see the difference, compare equation (2.3) with (2.4) against equations (C.11), (C.12) with (C.2) in appendix C. Further, from equations (2.3), (2.4) and appendix A one has

$$\begin{aligned} -\,m^2_{\phi_{H,0}}(q^2)\Big|_{\mathcal{B}} &= \frac{-1}{4} f_t^2 \mu^\epsilon \sum_{k_1,k_2\in\mathbb{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{(p+q)^2}{\left((p+q)^2 + \frac{4\left(k_1+\frac{1}{2}\right)^2}{R^2}\right)\left(p^2 + \frac{4\left(k_1+\frac{1}{2}\right)^2}{R^2}\right)} \\ &= \frac{-1}{4} f_t^2 \frac{\kappa_\epsilon}{(4\pi R)^2} \int_0^1 dx \left[\frac{2-\epsilon/2}{\pi} \mathcal{J}_2\left[\frac{1}{2},\frac{1}{2},c\right] + q^2 R^2 (1-x)^2 \mathcal{J}_1\left[\frac{1}{2},\frac{1}{2},c\right]\right] \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} -\,m^2_{\phi_{H,0}}(q^2)\Big|_{\mathcal{F}} &= \frac{1}{4} f_t^2 \mu^\epsilon \sum_{k_1,k_2\in\mathbb{Z}} \int \frac{d^d p}{(2\pi)^d} \frac{p.(p+q)}{\left((p+q)^2 + \frac{4k_2^2}{R^2}\right)\left(p^2 + \frac{4k_1^2}{R^2}\right)} \\ &= \frac{1}{4} f_t^2 \frac{\kappa_\epsilon}{(4\pi R)^2} \int_0^1 dx \left[\frac{2-\epsilon/2}{\pi} \mathcal{J}_2[0,0,c] + q^2 R^2 x(x-1) \mathcal{J}_1[0,0,c]\right] \end{aligned} \quad (2.6)$$

with the notations $\kappa_\epsilon \equiv (2\pi R\,\mu)^\epsilon$ and

$$\begin{aligned} \mathcal{J}_j[c_1,c_2,c] &\equiv \sum_{k_1,k_2\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{j-\epsilon/2}} e^{-\pi t (c + a_1(k_1+c_1)^2 + a_2(n_2+c_2)^2)}, \qquad j=1,2; \quad a_{1,2}\,, c>0\,; \\ & a_1 \equiv 4(1-x)\,, \qquad a_2 \equiv 4\,x\,, \qquad c \equiv x(1-x) q^2 R^2\,. \end{aligned} \quad (2.7)$$

The functions $\mathcal{J}_{1,2}$ are evaluated in appendix B. The total one-loop correction to the mass $m^2_{\phi_{H,0}}(q^2)$ is the sum of the two contributions in (2.5), (2.6). Note that only $\mathcal{J}_2$ contributes to $m^2_{\phi_{H,0}}(0)$, and this may be seen by formally setting $q^2=0$ in the second line of (2.5) and (2.6).

The quantities $\mathcal{J}_1$ and $\mathcal{J}_2$ have each a divergent and a finite part. Let us first discuss their divergent part which contributes to $m_{\phi_{H,0}}$. Even though in $\mathcal{J}_{1,2}$ one sums over the whole set $\mathbb{Z}$ of integers, and no finite set of levels (i.e. modes) is excluded from their infinite sums, their integrals are still UV divergent (i.e. at $t\to 0$). One shows that $\mathcal{J}_1$ and $\mathcal{J}_2$ have the behaviour

$$\mathcal{J}_j[c_1,c_2,c] = \left[\frac{2}{\epsilon}\right] \frac{(-\pi c)^j}{j\,\sqrt{a_1 a_2}} + \mathcal{O}(\epsilon^0)\,, \qquad j=1,2\,. \quad (2.8)$$

Although the divergences in $\mathcal{J}_1$ and $\mathcal{J}_2$ depend on $c\sim q^2R^2$, they are independent of

[6] We use $\sum_{k_1\geq 0} h\left(\left(k_1+\frac{1}{2}\right)^2\right) = \frac{1}{2}\sum_{k_1\in\mathbb{Z}} h\left(\left(k_1+\frac{1}{2}\right)^2\right)$ and $\sum_{k_1\geq 0} \frac{1}{2^{\delta_{k_1,0}}} h\left(k_1^2\right) = \frac{1}{2}\sum_{k_1\in\mathbb{Z}} h\left(k_1^2\right)$, h is an arbitrary function; this step is possible if Kaluza-Klein masses have special values (2.4), but not for those in (C.2).

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the coefficients c_1 and c_2 which here are fixed to 0 and 1/2 by the initial orbifold parity conditions on the 5D fields. Note the divergences of $\mathcal{J}_{1,2}$ are only "seen" for non-zero[7] $c \sim q^2 R^2$.

Let us consider now the finite part of $\mathcal{J}_{1,2}$. For simplicity we consider the case $c \ll 1$ respected under the sufficient assumption $q^2 \ll 1/R^2$. Equation (B.4) in appendix B gives, ignoring $\mathcal{O}(c)$ terms

$$\begin{aligned}
\mathcal{J}_1[0,0,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \ln\left[4\pi e^{-\gamma}|\eta(i\,u)|^4 \frac{1}{a_2}\right] - \ln(\pi e^{\gamma} c) \\
\mathcal{J}_1\left[\frac{1}{2},\frac{1}{2},c\ll 1\right] &= \frac{\pi c}{\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \ln\left|\frac{\vartheta_1\left(\frac{1}{2} - iu/2|iu\right)}{\eta(i\,u)} e^{-\pi u/4}\right|^2 \qquad u \equiv \left[\frac{a_1}{a_2}\right]^{\frac{1}{2}}
\end{aligned} \tag{2.9}$$

In a similar way one shows (see equation (B.7))

$$\begin{aligned}
\mathcal{J}_2[0,0,c\ll 1] &= \frac{-\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] + \frac{\pi^2 a_1 u}{45} + \\
&\quad + \frac{a_2}{\pi}\sum_{n\in\mathbb{Z}}\left[\mathrm{Li}_3\left(e^{-2\pi u|n|}\right) + 2\pi u|n|\mathrm{Li}_2\left(e^{-2\pi u|n|}\right)\right] \\
\mathcal{J}_2\left[\frac{1}{2},\frac{1}{2},c\ll 1\right] &= \frac{-\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \frac{7\pi^2 a_1 u}{360} + \\
&\quad + \frac{a_2}{\pi}\sum_{n\in\mathbb{Z}}\left[\mathrm{Li}_3\left(-e^{-2\pi u|n+\frac{1}{2}|}\right) + 2\pi u\left|n+\frac{1}{2}\right|\mathrm{Li}_2\left(-e^{-2\pi u|n+\frac{1}{2}|}\right)\right]
\end{aligned} \tag{2.10}$$

where again $\mathcal{O}(c)$ terms were neglected. For results without the restriction $c \ll 1$ see appendix B. The divergences in $\mathcal{J}_1$ and $\mathcal{J}_2$ in the last two sets of equations are in agreement with the general case of (2.8) which does not have the restriction $c \ll 1$. Equations (2.5), (2.6) give the total contribution

$$\begin{aligned}
-\, m^2_{\phi_{H,0}}(q^2) &= \frac{f_t^2}{32\pi^3 R^2}\int_0^1 dx\left[\mathcal{J}_2[0,0,c] - \mathcal{J}_2\left[\frac{1}{2},\frac{1}{2},c\right]\right] + \\
&\quad + \frac{f_t^2}{64\,\pi^2 R^2}\,(q^2 R^2)\int_0^1 dx\left[x(x-1)\mathcal{J}_1[0,0,c] - (1-x)^2\mathcal{J}_1\left[\frac{1}{2},\frac{1}{2},c\right]\right]
\end{aligned} \tag{2.11}$$

Note the presence of the factor q^2R^2 in front of the last integral. Therefore, if one studies only $m^2_{\phi_h,0}(0)$ the second line in (2.11) is absent. Further, if $c \sim q^2R^2 \ll 1$ the finite contributions from each $\mathcal{J}_1$ are suppressed, but the first term (divergent) in each $\mathcal{J}_1$ in (2.9), must be included in (2.11). Alternatively, one can use $\mathcal{J}_1$ of equation (2.9) and with $\mathcal{J}_2$ to order $\mathcal{O}(c^2)$ derived from appendix B or more generally, the full expressions given there.

The divergence of $\mathcal{J}_2$ in (2.10) or more generally (2.8) cancels out in $m^2_{\phi_{H,0}}$ because $\mathcal{J}_2[0,0,c]$ and $\mathcal{J}_2[1/2,1/2,c]$ have equal coefficient in equation (2.11). This cancellation is ensured by the equal number of bosonic and fermionic degrees of freedom, a remnant of initial 5D supersymmetry; in this $c^2 \sim q^4R^4$ plays a regularisation role. For $q^2 \to 0$ the

[7] If either $c_{1,2}$ are non-integers $\mathcal{J}_{1,2}$ are well defined even for $c = 0$; then the divergences of $\mathcal{J}_{1,2}$ are not "seen".

one-loop finite contribution $m^2_{\phi_H,0}(0)$ is due to $\mathcal{J}_2$ only and the result agrees numerically with that in [5] for a vanishing gauge coupling. More generally, from equation (2.11)

$$m^2_{\phi_{H,0}}(q^2) = m^2_{\phi_{H,0}}(0) - \frac{f_t^2}{2^{11}}\frac{1}{\epsilon}q^4R^2 + \frac{1}{R^2}\mathcal{O}(q^2R^2) \tag{2.12}$$

with $m^2_{\phi_{H,0}}(0)$ given by the first line in (2.11) with replacements[8] (2.10). Note that the above divergence q^4R^2/ϵ due to $\mathcal{J}_1$'s is of *similar form* to that cancelled by supersymmetry in the difference $\mathcal{J}_2[0,0,c] - \mathcal{J}_2[1/2,1/2,c]$ in equation (2.11) (see the terms in equations (2.10) proportional to $c^2/\epsilon \sim q^4R^2/\epsilon$). One notices the presence of a power four of the momentum scale in the contribution in (2.12). The emergence of the divergence in (2.12) shows that higher (dimension) derivative operators are required as one-loop counterterms for $m^2_{\phi_{H,0}}(q^2)$. The one-loop counterterm is[9] a "brane" $N=1$ interaction[10]

$$\begin{aligned}\int d^4x dx_5 \int d^2\theta d^2\overline{\theta}\delta(x_5)\lambda^2 H^\dagger \Box H &\sim f_t^2 \int d^4x R^2 \sum_{n,p\geq 0} \phi^\dagger_{H,n}\Box^2\phi_{H,p} \\ &\sim f_t^2 \int d^4x R^2 \phi^\dagger_{H,0}\Box^2\phi_{H,0} + \cdots \end{aligned} \tag{2.14}$$

where we used that $\lambda^2 = (\pi R)^3 f_t^2$. This interaction recovers in momentum space the one-loop correction in equation (2.12). If the compactification scale $1/R$ is high the suppression of the counterterm is important, below this scale the model is essentially four-dimensional and $m^2_{\phi_{H,0}}(q^2 \ll 1/R^2) \approx m^2_{\phi_{H,0}}(0) \sim 1/R^2$. However, this behaviour changes significantly for models with a "large" radius R or when $q^2R^2 \geq 1$. The presence of a higher derivative operator with an arbitrary (finite) coefficient is relevant for physics at $q^2 \geq 1/R^2$ and can be regarded as an effect of the non-renormalisability of the initial 5D theory. It can also affect the predictive power of such models. Finally, $m^2_{\phi_{H,0}}(q'^2) - m^2_{\phi_{H,0}}(q^2)$ can easily be evaluated using $\mathcal{J}_{1,2}$ of equations (2.9), (2.10) or their generalisations (B.1), (B.5), to provide the behaviour of the (mass)^2 under UV scaling of the momenta, with applications to phenomenology.

The pole in (2.12) due to $\mathcal{J}_1$ is just a result of compactification: it would not be present if we included only a fixed, finite number of levels, even if this were done such as to respect their ($N=2$) multiplet structure. To see this consider the situation when the sums over the Kaluza-Klein levels which enter $\mathcal{J}_1[1/2,1/2,c]$ and $\mathcal{J}_1[0,0,c]$ are restricted to a finite number of modes, hereafter called s_i and s'_i with $i=1,2$. The equivalent of $\mathcal{J}_1$ defined

[8]The series of Li_3 in equation (2.11) due to $\mathcal{J}_2$'s in (2.10) can be integrated analytically, using [24] (Corollary 5.8 for $n=1$).

[9]In (2.13), (2.14) the mass dimension of the superfields H, H^c is $3/2$.

[10]One could also think of a $N=2$ "bulk" counterterm to generate q^4R^2/ϵ required in (2.12)

$$\int d^4x dx_5 \int d^2\theta d^2\overline{\theta} R^2 \left(H^\dagger\Box H + H^{c\dagger}\Box H^c\right) \sim \int d^4x R^2 \phi^\dagger_{H,0}\Box^2\phi_{H,0} + \cdots \tag{2.13}$$

but this is not possible since the H^c dependent term is not generated, (H^c has no Yukawa coupling, see equation (2.2)).

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in (2.7) and present in (2.5), (2.6) is then

$$\mathcal{J}_1^*\left[\frac{1}{2},\frac{1}{2},c\right] \equiv 4\sum_{0\le n_i\le s_i}\int_0^\infty \frac{dt}{t^{1-\epsilon/2}}e^{-\pi t[a_1(n_1+\frac{1}{2})^2+a_2(n_2+\frac{1}{2})^2+c]} = \frac{2}{\epsilon}(4\,s_1s_2)+\mathcal{O}(\epsilon^0)$$

$$\mathcal{J}_1^*[0,0,c] \equiv 4\sum_{0\le n_i\le s_i'}\frac{1}{2^{\delta_{n_1,0}}}\frac{1}{2^{\delta_{n_2,0}}}\int_0^\infty \frac{dt}{t^{1-\epsilon/2}}e^{-\pi t[a_1n_1^2+a_2n_2^2+c]} = \\ = \frac{2}{\epsilon}(2s_1'-1)(2s_2'-1)+\mathcal{O}(\epsilon^0) \qquad (2.15)$$

Therefore if the KK towers were restricted to a finite level the divergences in $\mathcal{J}_1^*$ are independent of $c\sim q^2R^2$ which was not the case of $\mathcal{J}_1$ in equation (2.8). This shows that the limits $s_i\to\infty$ (or $s_i'\to\infty$) and $\epsilon\to 0$ of $\mathcal{J}_1^*$ do not commute and lead to different divergences.[11] From equations (2.3), (2.5), (2.6), (2.7), (2.15) we find the contribution of $\mathcal{J}_1^*$ to $m^2_{\phi_{H,0}}(q^2)$

$$-\,m^2_{\phi_{H,0}}(q^2)\Big|_{\mathcal{J}_1^*} = \frac{1}{4}\frac{f_t^2}{(4\pi R)^2}(q^2R^2)\int_0^1 dx\left[x(x-1)\mathcal{J}_1^*[0,0,c]-(1-x)^2\mathcal{J}_1^*\left[\frac{1}{2},\frac{1}{2},c\right]+\mathcal{O}(\epsilon^0)\right] \\ = -\frac{f_t^2}{192\pi^2}\left[\frac{q^2}{\epsilon}\right]\Big((2s_1'-1)(2s_2'-1)+8s_1s_2\Big)+\mathcal{O}(\epsilon^0) \qquad (2.16)$$

The divergence q^4R^2/ϵ in equation (2.12) has changed into q^2/ϵ, equation (2.16) in the case of including only a finite number of modes in each $\mathcal{J}_1$. The latter divergence is the expected wavefunction renormalisation of the chiral superfield of the zero-mode scalar. Therefore the presence of higher dimension (derivative) operators responsible for q^4R^2/ϵ is a consequence of summing over infinitely many modes[12] (or "large enough" a number) and thus of the compactification of the initial 5D theory.

3. Further remarks and conclusions.

In a minimal 5D $N=1$ supersymmetric model compactified on $S_1/(Z_2\times Z_2')$ with a brane-localised superpotential it was shown that higher derivative operators emerge as one-loop counterterms to the mass of the zero-mode scalar field. This finding is relevant for more realistic orbifold models [1]–[11], where this scalar field is usually identified with the Higgs boson, and our technical results can easily be used in such models. It was shown that the emergence of such operators as counterterms is a direct effect of compactification, i.e. of the integration of infinitely many modes of the initial theory. The presence of such counterterms is not surprising in the end, if we remember that the 5D theory although supersymmetric, is nevertheless non-renormalisable, but such effect was so far overlooked.

[11] Though their finite part is the same in the continuum limit, when an extra divergence in s_i also emerges in (2.15).

[12] An observation is in place here: one may argue that in general radiative corrections from massive modes "winding" around compact dimension(s) can only induce finite loop corrections. This argument ignores the degeneracy of the modes and thus is not always correct (this is the case where we have *two* (infinite) Kaluza-Klein sums).

The result was obtained using a DR regularisation scheme for the 4D momentum integrals. While suppressed at low scales $q^2 \ll 1/R^2$, where the theory is essentially four dimensional, the operators play an increasingly important role as we scale the momentum q^2 to UV values $q^2 \gg 1/R^2$. Our results can also be used to investigate $m^2_{\phi_{H,0}}(q^2)$ behaviour at $q^2 \sim 1/R^2$ and under the UV scaling of q^2, giving a quartic dependence on the UV scaling factor. Such operators may also appear in a similar way in the one-loop corrected Yukawa coupling. The situation is similar to the case of radiative corrections to the gauge couplings in models with *two* compact dimensions, where higher derivative operators emerge as one-loop counterterms in a very similar way [13].

A natural question is whether such operators can be generated for other one dimensional orbifolds. This depends on the orbifold type and the interaction considered. The technical reason for their presence in $S_1/(Z_2 \times Z_2')$ was the (degeneracy of the levels due to the) existence of *two* Kaluza-Klein sums in the one-loop two-point Green function. This is in turn due to the brane-localisation of interaction (2.2) with *all fields* living in the "bulk". In momentum space, with external legs of the diagrams fixed to zero-level ($\phi_{H,0}$), this lead to two Kaluza-Klein sums in front of the loop-integral of the correction to the $(\text{mass})^2$; their evaluation generated the higher derivative operators. Similar arguments apply if one considered the *same* brane-localised interaction in S_1/Z_2 orbifolds and then higher derivative operators may emerge as one-loop counterterms to the mass of the zero-mode scalar. However, if the brane-localised interaction (2.2) had only *one* "bulk" field, with the other two genuine 4D "brane" fields, overall only *one* Kaluza-Klein sum would be present in the correction to the mass of a (brane) scalar field. In that case such operators are not generated at one-loop, but at higher loops they can be present. This last observation also applies to a bulk interaction rather than the brane-localised interaction discussed above.

There is another, simpler way to understand these statements, based on purely dimensional arguments applied to the localised interaction $\lambda\, Q\, U\, H$ of (2.2) and the general form of the localised counterterm $(\lambda^2)^n H^\dagger \Box H$ of (2.14), where we used that the latter must not depend on R. If all these fields are bulk fields, λ has mass dimension $[\lambda] = -3/2$ giving $n = 1$ i.e. the counterterm appears at one-loop, in agreement with the findings of the paper and with equation (2.14). However, if the interaction has two genuine brane fields and one bulk field, $[\lambda] = -1/2$. Then the local counterterm giving q^4 dependence has, if H is a brane field, $n = 2$, thus it is generated at the two-loop level, as already argued above. If H is a bulk field instead, dimensional arguments give that $n = 3$, thus such counterterm arises at three-loop only. Similar considerations can be made for the bulk interactions using that the gauge coupling also has mass dimension $-1/2$. These observations can be used when building higher dimensional models, to assess the importance and avoid the presence of higher derivative operators (counterterms) at small number of loops.

Such operators are relevant for phenomenology. The presence of higher derivative operators with arbitrary coefficients which are new parameters in the theory (depending on its UV completion) may affect the predictive power of the models, particularly if the compactification scale is small. This can be important for the phenomenology of 5D orbifold models with a large extra-dimension, although only a model-by-model study can provide a definite answer to the importance of the effects of such operators.

The above analysis only discussed the case of a "local" interaction at one fixed point of the orbifold $S_1/(Z_2 \times Z_2')$. However, the "global" structure of the orbifold can involve additional interactions (at the other fixed points), not considered here. Such "global" structure can only be seen by studying "overlapping" interactions originating from different fixed points, and that may reveal new effects. This involves calculations beyond one-loop order and is beyond the purpose of this work.

Finally, since field theory orbifolds are now extensively used for model building, we provided in appendix C a generalisation of the method used in the text for computing series of divergent loop-integrals of the radiative corrections generic in orbifold compactifications. The method applies for a more general Kaluza-Klein spectrum than that used in the text.[13] The approach pays special attention to regularisation subtleties within a DR scheme for the momentum integrals, to show a far richer set of divergences than that of the specific example discussed above. We hope this will help a careful investigation of more realistic models.

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A. One-loop integrals in DR.

Calculations in DR (see for example [25]) give that ($d = 4 - \epsilon$, $\epsilon \to 0$)

$$\mathcal{K}_1 \equiv \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu}{((p+q)^2 + m_2^2)(p^2 + m_1^2)} = \frac{-q_\mu}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx x \Gamma\left[2 - \frac{d}{2}\right] \left[L(x, q^2, m_{1,2})\right]^{\frac{d}{2}-2}$$

$$\mathcal{K}_2 \equiv \int \frac{d^d p}{(2\pi)^d} \frac{1}{((p+q)^2 + m_2^2)(p^2 + m_1^2)} = \frac{1}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx \Gamma\left[2 - \frac{d}{2}\right] \left[L(x, q^2, m_{1,2})\right]^{\frac{d}{2}-2}$$

$$\mathcal{K}_3 \equiv \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu p_\nu}{((p+q)^2 + m_2^2)(p^2 + m_1^2)} = \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\delta_{\mu\nu}}{2} \int_0^1 dx \Gamma\left[1 - \frac{d}{2}\right] \left[L(x, q^2, m_{1,2})\right]^{\frac{d}{2}-1} + \\ + \frac{1}{(4\pi)^{\frac{d}{2}}} q_\mu q_\nu \int_0^1 dx x^2 \Gamma\left[2 - \frac{d}{2}\right] \left[L(x, q^2, m_{1,2})\right]^{\frac{d}{2}-2}$$

where

$$L(x, q^2, m_{1,2}) \equiv x(1-x) q^2 + x m_2^2 + (1-x) m_1^2 \,. \tag{A.1}$$

With the notation $\mathcal{L} \equiv R^2 L(x, q^2, m_{1,2})$ and $\sum_\mu \delta_{\mu\mu} = d$ one finds:

$$\mathcal{K}_b \equiv \int \frac{d^d p}{(2\pi)^d} \frac{(p+q)^2}{((p+q)^2 + m_2^2)(p^2 + m_1^2)}$$

[13] See for example the spectrum in equation (C.2) in the appendix and compare against that in equation (2.4).

$$= \frac{(R^2)^{1-\frac{d}{2}}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx \left[\frac{d}{2} \mathcal{L}^{\frac{d}{2}-1} \Gamma\left[1-\frac{d}{2}\right] + q^2 R^2 (1-x)^2 \mathcal{L}^{d/2-2} \Gamma\left[2-\frac{d}{2}\right]\right] \tag{A.2}$$

$$\mathcal{K}_f \equiv \int \frac{d^d p}{(2\pi)^d} \frac{p^2 + p.q}{((p+q)^2 + m_2^2)(p^2+m_1^2)}$$

$$= \frac{(R^2)^{1-\frac{d}{2}}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx \left[\frac{d}{2} \mathcal{L}^{\frac{d}{2}-1} \Gamma\left[1-\frac{d}{2}\right] + q^2 R^2 \, x\,(x-1) \mathcal{L}^{\frac{d}{2}-2} \Gamma\left[2-\frac{d}{2}\right]\right] \tag{A.3}$$

Using the identity $\Gamma[s]\,(\pi\mathcal{L})^{-s} = \int_0^\infty dt\, t^{s-1}\, e^{-\pi\mathcal{L}\,t}$ one then finds equations (2.5),(2.6) in the text.

B. Series of divergent integrals in DR.

Following calculations in [26] (see also [27]), one shows that for $c, a_1, a_2 > 0$ and real c_1, c_2

$$\mathcal{J}_1[c_1,c_2,c] \equiv \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{1-\epsilon/2}} e^{-\pi t [c + a_1(n_1+c_1)^2 + a_2(n_2+c_2)^2]}$$

$$= \frac{\pi c}{\sqrt{a_1 a_2}} \left[\frac{-2}{\epsilon}\right] + 2\pi \left[\frac{a_1}{a_2}\right]^{\frac{1}{2}} \left[\frac{1}{6} + \Delta_{c_1}^2 - \left(\frac{c}{a_1} + \Delta_{c_1}^2\right)^{\frac{1}{2}}\right]$$

$$+ \frac{\pi c}{\sqrt{a_1 a_2}} \ln\left[4\pi a_1 e^{\gamma + \psi(\Delta_{c_1}) + \psi(-\Delta_{c_1})}\right] - \sum_{n_1\in\mathbb{Z}} \ln\left|1 - e^{\frac{-2\pi}{\sqrt{a_2}}[c + a_1(n_1+c_1)^2]^{\frac{1}{2}} + 2i\pi c_2}\right|^2$$

$$+ \left[\pi\frac{a_1}{a_2}\right]^{\frac{1}{2}} \sum_{p\geq 1}^{\infty} \frac{\Gamma[p+\frac{1}{2}]}{(p+1)!} \left[\frac{-c}{a_1}\right]^{p+1} \left[\zeta[2p+1, 1+\Delta_{c_1}] + \zeta[2p+1, 1-\Delta_{c_1}]\right] \tag{B.1}$$

$\zeta[z,q]$ is the Hurwitz Zeta function, $\psi(x) = d/dx\, \ln\Gamma[x]$. The result only depends on the fractional part of c_i $(i=1,2)$ defined by $\Delta_{c_i} \equiv c_i - [c_i]$ with $0 \leq \Delta_{c_i} < 1$ and $[c_i] \in \mathbb{Z}$. The exclusion of a finite set of modes/levels (such as a zero mode, etc) from the two sums in the definition of $\mathcal{J}_1$ would however bring in a dependence of $\mathcal{J}_1$ on c_i (rather than Δ_{c_i}) and also additional poles in ϵ. With $u \equiv \sqrt{a_1/a_2}$ one has

$$\mathcal{J}_1[c_1,c_2,c \ll 1] = \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{1-\epsilon/2}}\, e^{-\pi\, t\,[\,c + a_1(n_1+c_1)^2 + a_2(n_2+c_2)^2]} \tag{B.2}$$

$$= \frac{\pi c}{\sqrt{a_1 a_2}} \left[\frac{-2}{\epsilon}\right] - \ln\left|\frac{\vartheta_1(c_2 - iu\,c_1|iu)}{(c_2 - iuc_1)\eta(i\,u)} e^{-\pi u\, c_1^2}\right|^2 - \ln\left[\frac{(c + a_1 c_1^2 + a_2 c_2^2)}{a_2}\right]$$

Above we only kept the term proportional to c/ϵ because the limits $c \to 0$ and $\epsilon \to 0$ do not commute and neglected $\mathcal{O}(c)$ terms; we used

$$\vartheta_1(z|\tau) \equiv -i \sum_{n\in\mathbb{Z}} (-1)^n\, e^{i\pi\tau(n+\frac{1}{2})^2}\, e^{(2n+1)i\pi z}, \qquad \eta(\tau) \equiv e^{i\pi\tau/12} \prod_{n\geq 1} (1 - e^{2i\pi\tau n}) \tag{B.3}$$

for the Jacobi Theta function ϑ_1 and Dedekind function η, respectively.

For the particular cases encountered in the text, equation (B.2) gives

$$\begin{aligned}
\mathcal{J}_1[0,0,c\ll 1] &= \frac{\pi c}{\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \ln\left[4\pi e^{-\gamma}|\eta(iu)|^4 \frac{1}{a_2}\right] - \ln(\pi e^{\gamma} c) \\
\mathcal{J}_1\left[\frac{1}{2},\frac{1}{2},c\ll 1\right] &= \frac{\pi c}{\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \ln\left|\frac{\vartheta_1(\frac{1}{2}-iu/2|iu)}{\eta(iu)}\right|^2 + \frac{\pi}{2}u\,, \qquad u\equiv\sqrt{\frac{a_1}{a_2}}
\end{aligned} \tag{B.4}$$

used in equation (2.9).

A similar calculation gives that, for $c, a_1, a_2 > 0$ and c_1, c_2 real

$$\begin{aligned}
\mathcal{J}_2[c_1,c_2,c] &\equiv \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{2-\epsilon/2}} e^{-\pi t[c+a_1(n_1+c_1)^2+a_2(n_2+c_2)^2]} \\
&= -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] + \frac{\pi^2 a_1^{3/2}}{3 a_2^{\frac{1}{2}}}\left[\frac{1}{15} - 2\Delta_{c_1}^2\left(1+\Delta_{c_1}^2+\frac{3c}{a_1}\right) - \frac{c}{a_1} + 4\left(\frac{c}{a_1}+\Delta_{c_1}^2\right)^{\frac{3}{2}}\right] + \\
&\quad + \sum_{n_1\in\mathbb{Z}} \left[a_2(c+a_1(n_1+c_1)^2)\right]^{\frac{1}{2}} \mathrm{Li}_2\left(e^{-2\pi[(c/a_2+a_1(n_1+c_1)^2/a_2)^{\frac{1}{2}}-ic_2]}\right) + \\
&\quad + \frac{a_2}{2\pi}\sum_{n_1\in\mathbb{Z}} \mathrm{Li}_3\left(e^{-2\pi[(c/a_2+a_1(n_1+c_1)^2/a_2)^{\frac{1}{2}}-ic_2]}\right) - \\
&\quad - \frac{\pi^2 c^2}{2\sqrt{a_1 a_2}} \ln\left[4\pi e^{\gamma+\psi(\Delta_{c_1})+\psi(-\Delta_{c_1})}\right] + \\
&\quad + \frac{\pi^{3/2} c^2}{\sqrt{a_1 a_2}} \sum_{p\geq 1}^{\infty} \frac{\Gamma\left[p+\frac{1}{2}\right]}{(p+2)!}\left[\frac{-c}{a_1}\right]^p [\zeta[2p+1,1+\Delta_{c_1}]+\zeta[2p+1,1-\Delta_{c_1}]] + \text{c.c.}
\end{aligned} \tag{B.5}$$

where "c.c." only applies to the PolyLogarithm functions Li_2 and Li_3. Similar to the previous case, we introduced the fractional part of c_i defined as $\Delta_{c_i} \equiv c_i - [c_i]$, $0 \leq \Delta_{c_i} < 1$ and $[c_i] \in \mathbb{Z}$.

Equation (B.5) gives:

$$\begin{aligned}
\mathcal{J}_2[c_1,c_2,c\ll 1] &= \sum_{n_1,n_2\in\mathbb{Z}} \int_0^\infty \frac{dt}{t^{2-\epsilon/2}} e^{-\pi t[c+a_1(n_1+c_1)^2+a_2(n_2+c_2)^2]} \\
&= -\frac{\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] + \frac{\pi^2 a_1}{3}\left[\frac{a_1}{a_2}\right]^{\frac{1}{2}}\left[\frac{1}{15} - 2\Delta_{c_1}^2(1-\Delta_{c_1})^2\right] + \\
&\quad + \left[\sqrt{a_1 a_2}\sum_{n\in\mathbb{Z}} |n+c_1|\,\mathrm{Li}_2(e^{-2\pi i z}) + \frac{a_2}{2\pi}\sum_{n\in\mathbb{Z}} \mathrm{Li}_3(e^{-2\pi i z}) + \text{c.c.}\right]
\end{aligned} \tag{B.6}$$

with $c, a_1, a_2 > 0$ and $z = c_2 - i\,(a_1/a_2)^{\frac{1}{2}}|n+c_1|$. We kept the term proportional to c^2/ϵ because the limits $c \to 0$ and $\epsilon \to 0$ do not commute. Equation (B.6) finally gives

$$\mathcal{J}_2[0,0,c\ll 1] = \frac{-\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] + \frac{\pi^2 a_1 u}{45} + \frac{a_2}{\pi}\sum_{n\in\mathbb{Z}}\left[\mathrm{Li}_3(e^{-2\pi\, u\,|n|}) + 2\pi u|n|\,\mathrm{Li}_2(e^{-2\pi u|n|})\right]$$

$$\mathcal{J}_2\left[\frac{1}{2},\frac{1}{2},c\ll 1\right] = \frac{-\pi^2 c^2}{2\sqrt{a_1 a_2}}\left[\frac{-2}{\epsilon}\right] - \frac{7\pi^2 a_1 u}{360} + $$
$$+\frac{a_2}{\pi}\sum_{n\in\mathbb{Z}}\left[\mathrm{Li}_3(-e^{-2\pi u\,|n+\frac{1}{2}|}) + 2\pi u\left|n+\frac{1}{2}\right|\mathrm{Li}_2(-e^{-2\pi u|n+\frac{1}{2}|})\right] \tag{B.7}$$

with

$$u \equiv \sqrt{\frac{a_1}{a_2}}$$

The two $\mathcal{J}_2$ in (B.7) were used in equation (2.10).

C. General evaluation of series of (divergent) loop-integrals using DR.

When computing radiative corrections in orbifold compactifications one has to sum infinitely many contributions from divergent loop integrals (of 4D Feynman diagrams), of general structure shown in equation (C.1). Here we show a careful evaluation of this expression which generalises that encountered in the text equation (2.3) with (2.4). With minimal changes this can be applied to one-loop calculations in $S_1/(Z_2\times Z_2')$, in some S_1/Z_2 models at one-loop[14] [7] and also in two dimensional compactifications, given the presence of a double KK sum in (C.1). The technique can also be used in cases with one (rather than two) infinite sums. The method is useful for an easy evaluation of the overall divergence of the final result.[15] Therefore we shall compute

$$\mathcal{H} \equiv \sum_{k_1\geq 0}\sum_{k_2\geq 0}\int \frac{d^d p}{(2\pi)^d}\frac{\alpha p^2 + \beta(p.q) + \gamma q^2 + \delta}{[(q+p)^2 + m_{k_2}^2]^n\,[p^2 + m_{k_1}^2]^m}\,. \tag{C.1}$$

Here q is an (euclidean) external momenta. The loop integrals are regularised in DR with $d = 4-\epsilon$, $\epsilon \to 0$. No additional regulator is needed for the infinite sum(s). $m_{k_{1,2}}$ are KK masses in the loops, and the sums are restricted to positive modes only due to (orbifold) parity constraints. α, β, γ are arbitrary, dimensionless parameters while δ has dimension of $(\text{mass})^2$. Here we take $n, m \geq 1$ and the most general structure for the masses of the KK states

$$m_{k_1} = \frac{u}{R}(k_1 + c_1)\,, \qquad m_{k_2} = \frac{1}{R}(k_2 + c_2)\,. \tag{C.2}$$

For generality we introduced u an arbitrary positive constant (dimensionless). We denote $\mathcal{H}_{k_1,k_2}$ the momentum integral in (C.1). Standard integration techniques give [25]

$$\mathcal{H}_{k_1,k_2} = \frac{1}{(4\pi)^{\frac{d}{2}}}\frac{1}{\Gamma[n]\Gamma[m]}\left\{\int_0^1 dx\, x^{n-1}(1-x)^{m-1}\times\right.$$
$$\times\left[\left(\delta + q^2(\gamma - x\beta + x^2\alpha)\right)\Gamma\left[n+m-\frac{d}{2}\right]L^{\frac{d}{2}-n-m} + \right.$$

[14] Our regularisation can be used in models where series like (C.1) appear at two-loop only [3], equation (4.39).

[15] See also [28] for a related calculation.

$$+\alpha\frac{d}{2}\Gamma\left[n+m-\frac{d}{2}-1\right]L^{\frac{d}{2}-n-m+1}\Big]\Big]\Big\}$$

$$L\equiv x(1-x)q^2+(1-x)m_{k_1}^2+xm_{k_2}^2 \tag{C.3}$$

UV divergences in $\mathcal{H}$ arise from the two Gamma functions in the square brackets above and possibly from the integrals over x. With $d=4-\epsilon$ poles in ϵ are present only if $n+m\leq 3$. To find the overall divergence of $\mathcal{H}$ one must evaluate the infinite sums below, equation (C.4) with $s=\epsilon/2$; $\epsilon/2-1$ to order $\mathcal{O}(\epsilon^0)$. To find the finite part of $\mathcal{H}$ one further needs the $\mathcal{O}(\epsilon)$ terms. We restrict the calculation to $\mathcal{O}(\epsilon^0)$ only, to find the divergent part of $\mathcal{H}$, but the analysis is easily extended to find its finite part as well. From (C.1), (C.3) with (C.2) we conclude that we must evaluate

$$\sum_{k_1\geq 0}\sum_{k_2\geq 0}\left[c+\sigma_1(k_1+c_1)^2+\sigma_2\,(k_2+c_2)^2\right]^{-s},$$

$$\sigma_1\equiv u^2(1-x)\,,\qquad \sigma_2\equiv x\,,\qquad c\equiv x(1-x)q^2R^2 \tag{C.4}$$

for $s=\epsilon/2$ and $-1+\epsilon/2$ at $\mathcal{O}(\epsilon^0)$. The evaluation of (C.4) is known in the context of Epstein-like functions. Such sums are usually evaluated for the whole set $\mathbb{Z}$ of integers. However, in orbifold constructions parity constraints require one perform the sums for positive integers only, and with arbitrary $c_{1,2}$, which is a more difficult task. To evaluate (C.4) we rely on the general result for a one dimensional sum extensively studied in the literature [29], and then use the result in equation (C.4). We introduce

$$E_1[c;s;\sigma_1,c_1]\equiv\sum_{n_1\geq 0}\left[\sigma_1(n_1+c_1)^2+c\right]^{-s}. \tag{C.5}$$

This has the *asymptotic* expansion (see [29] for a proof)

$$E_1[c;s;\sigma_1,c_1]\approx c^{-s}\sum_{m\geq 0}\frac{\Gamma[s+m]}{m!\Gamma[s]}\left[\frac{-\sigma_1}{c}\right]^m\zeta[-2m,c_1]+\frac{c^{\frac{1}{2}-s}}{2}\left[\frac{\pi}{\sigma_1}\right]^{\frac{1}{2}}\frac{\Gamma\left[s-\frac{1}{2}\right]}{\Gamma[s]}+$$
$$+\frac{2\pi^s}{\Gamma[s]}\cos(2\pi c_1)\sigma_1^{-\frac{s}{2}-\frac{1}{4}}c^{-\frac{s}{2}+\frac{1}{4}}\sum_{n_1\geq 1}n_1^{s-\frac{1}{2}}K_{s-\frac{1}{2}}\left(2\pi n_1\left(\frac{c}{\sigma_1}\right)^{\frac{1}{2}}\right) \tag{C.6}$$

where $\zeta[q,a], a\neq 0,-1,-2,\ldots$ is the Hurwitz Zeta function, $\zeta[q,a]=\sum_{n\geq 0}(n+a)^{-q}$ for $Re(q)>1$. K is the usual modified Bessel function [30]. The singularities of E_1 can arise for specific values of s, from poles of the Gamma functions in the rhs of the first line in[16]

[16] An alternative to using (C.6) is to make a simple *binomial* expansion of the parenthesis under the sum in (C.5) in powers of $c/\sigma_1\leq 1$ and then just use the definition of Hurwitz ζ-function. This easily gives:

$$E_1[c;s;\sigma_1,c_1]\,=\,\sigma_1^{-s}\sum_{k\geq 0}\frac{\Gamma[k+s]}{k!\Gamma[s]}\left[\frac{-c}{\sigma_1}\right]^k\zeta[2k+2s,c_1]\,. \tag{C.7}$$

The singularities of E_1 given in (C.8) arise now from either the Gamma functions (if s is a negative integer or zero) and from the usual singularity of ζ-function at $2k+2s=1$, (if s is $\pm 1/2,-3/2,-5/2,-7/2,\cdots$).

equation (C.6). Using either equation (C.6) or (C.7) one can easily show that

$$\begin{aligned} E_1[c;-\epsilon;\sigma_1,c_1] &= \zeta[0,c_1]+\mathcal{O}(\epsilon) \\ E_1[c;-1-\epsilon,\sigma_1,c_1] &= \sigma_1\,\zeta[-2,c_1]+c\zeta[0,c_1]+\mathcal{O}(\epsilon) \\ E_1[c;-\epsilon-1/2;\sigma_1,c_1] &= -\frac{c}{4\sqrt{\sigma_1}}\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0) \\ E_1[c;-\epsilon-3/2;\sigma_1,c_1] &= -\frac{3c^2}{16\sqrt{\sigma_1}}\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0) \end{aligned} \tag{C.8}$$

where one can further use that $\zeta[0,x]=1/2-x$. These results will be used shortly to evaluate the double sums in (C.4).
Further, equation (C.6) gives, after the replacement $c\to c+\sigma_2(n_2+c_2)^2$ and a summation over n_2:

$$\begin{aligned} E_2[c;s;\sigma_1,\sigma_2;c_1,c_2] &\equiv \sum_{n_1\geq 0}\sum_{n_2\geq 0}\Big[\sigma_1(n_1+c_1)^2+\sigma_2(n_2+c_2)^2+c\Big]^{-s} \\ &\approx \frac{\sigma_2^{-s}}{\Gamma[s]}\sum_{m\geq 0}\frac{(-1)^m}{m!}\Gamma[s+m]\left[\frac{\sigma_1}{\sigma_2}\right]^m\zeta[-2m,c_1]E_1\left[\frac{c}{\sigma_2};s+m;1,c_2\right]+ \\ &\quad+\sigma_2^{\frac{1}{2}-s}\left[\frac{\pi}{\sigma_1}\right]^{\frac{1}{2}}\frac{\Gamma[s-\frac{1}{2}]}{2\Gamma[s]}E_1\left[\frac{c}{\sigma_2};s-\frac{1}{2};1,c_2\right]+\frac{2\pi^s}{\Gamma[s]}\cos(2\pi c_1)\sigma_1^{-\frac{s}{2}-\frac{1}{4}}\times \\ &\quad\times\sum_{n_1\geq 1}\sum_{n_2\geq 0}n_1^{s-\frac{1}{2}}\left[\sigma_2(n_2+c_2)^2+c\right]^{-\frac{s}{2}+\frac{1}{4}}\times \\ &\quad\times K_{s-\frac{1}{2}}\left(\frac{2\pi n_1}{\sigma_1^{1/2}}\left[\sigma_2(n_2+c_2)^2+c\right]^{\frac{1}{2}}\right) \end{aligned} \tag{C.9}$$

For an extended study of the properties of E_2 see [29].

Using now equations (C.8) in (C.9), one finally finds from (C.9) with $s=\epsilon/2$ or $s=-1+\epsilon/2$:

$$\begin{aligned} \mathcal{R}_0 &\equiv \sum_{n_1>0}\sum_{n_2>0}\left[\sigma_1(n_1+c_1)^2+\sigma_2(n_2+c_2)^2+c\right]^{-\epsilon/2} = -\frac{\pi}{4}\frac{c}{\sqrt{\sigma_1\sigma_2}}+\zeta[0,1+c_1]\zeta[0,1+c_2]+\mathcal{O}(\epsilon) \\ \mathcal{R}_1 &\equiv \sum_{n_1>0}\sum_{n_2>0}\left[\sigma_1(n_1+c_1)^2+\sigma_2(n_2+c_2)^2+c\right]^{1-\epsilon/2} \\ &= -\frac{\pi}{8}\frac{c^2}{\sqrt{\sigma_1\sigma_2}}+\frac{1}{3}\left(c_1+\frac{1}{2}\right)\left(c_2+\frac{1}{2}\right)\left[\sigma_1c_1(c_1+1)+\sigma_2c_2(c_2+1)+3c\right]+\mathcal{O}(\epsilon) \end{aligned} \tag{C.10}$$

which computes (C.4). From (C.1),(C.2), (C.3), (C.10) we finally find the overall divergence of $\mathcal{H}$

$$\begin{aligned} \mathcal{H}|_{n=m=1} &= \sum_{k_1\geq 0}\sum_{k_2\geq 0}\int\frac{d^dp}{(2\pi)^d}\frac{\alpha p^2+\beta(p.q)+\gamma q^2+\delta}{[(q+p)^2+(k_2+c_2)^2/R^2][p^2+(k_1+c_1)^2u^2/R^2]} \\ &= \frac{1}{(4\pi)^2}\frac{2}{\epsilon}\left[\delta-\frac{\alpha}{3R^2}\left(u^2(c_1-1)c_1+(c_2-1)c_2\right)+q^2\left(\gamma-\frac{\beta}{2}\right)\right]\left(c_1-\frac{1}{2}\right)\left(c_2-\frac{1}{2}\right)+ \end{aligned}$$

$$+\frac{1}{(4\pi)^2}\frac{2}{\epsilon}\left[-\frac{\pi^2}{32u}\delta\, q^2R^2-\frac{\pi^2}{2^9u}(2\alpha-8\beta+16\gamma)q^4R^2\right]+\mathcal{O}(\epsilon^0)\,,$$

$$\mathcal{H}|_{n+m=3}=-\frac{1}{(4\pi)^2}\frac{2}{\epsilon}\alpha\left[\frac{\pi^2}{32u}q^2R^2-\left(c_1-\frac{1}{2}\right)\left(c_2-\frac{1}{2}\right)\right]+\mathcal{O}(\epsilon^0) \qquad \text{(C.11)}$$

for arbitrary $\alpha,\beta,\gamma,\delta,c_1,c_2$ and $u>0$. In (C.11) we used that $n\geq 1, m\geq 1$. Note also a generalisation of the fermionic contribution presented in the text

$$\mathcal{F}\equiv\sum_{k_1\geq 0}\sum_{k_2\geq 0}\frac{1}{2^{\delta_{k_1,0}}}\frac{1}{2^{\delta_{k_2,0}}}\int\frac{d^dp}{(2\pi)^d}\frac{\alpha p^2+\beta(p.q)+\gamma q^2+\delta}{[(q+p)^2+(k_2+c_2)^2/R^2][p^2+(k_1+c_1)^2\,u^2/R^2]}$$
$$=\frac{1}{(4\pi)^2}\,\frac{2}{\epsilon}\left[\delta-\frac{\alpha}{6\,R^2}\left(1+u^2+2\,(c_1^2u^2+c_2^2)\right)+q^2\left(\gamma-\frac{\beta}{2}\right)\right]c_1c_2+$$
$$+\frac{1}{(4\pi)^2}\frac{2}{\epsilon}\left[-\frac{\pi^2}{32u}\delta q^2R^2-\frac{\pi^2}{2^9u}(2\,\alpha-8\beta+16\gamma)\,q^4R^2\right]+\mathcal{O}(\epsilon^0) \qquad \text{(C.12)}$$

which can be compared against (C.11).

In conclusion $\mathcal{H}$ of (C.11) has divergences which are both dependent and independent of q^2. The later are not all "seen" in the special cases c_1 or c_2 is equal to $1/2$ or 0 or when $\alpha=\delta=0$ (similar considerations apply to $\mathcal{F}$ of (C.12)). The q^2 or q^4-dependent divergences are associated with wavefunction renormalisation or higher dimension derivative operators and are absent only for special values of the parameters (e.g. $\delta=0$, $\gamma=\beta/2$ and respectively $2\alpha-8\beta+16\gamma=0$). One can also compute the $\mathcal{O}(\epsilon^0)$ terms in (C.11) following the same approach. Note that in some cases additional singularities in x (not in ϵ) may appear [25] from $\mathcal{O}(\epsilon^0)$ terms, from the integrals over x. The above results can be used in one and two dimensional compactifications while summing infinitely many Feynman diagrams, and provide a careful approach to computing their overall divergence.

As an application to the $S_1/(Z_2\times Z_2')$ orbifold, equation (C.11) with $u=1$, $\delta=0$, $\gamma=1$, $\beta=2$, $\alpha=1$, $c_1=c_2=1/2$, $R\to R/2$ gives, up to an overall sign, the contribution for bosons encountered in the text, equations (2.5). Similarly, equation (C.12) for $u=1$, $\delta=0$, $\gamma=0$, $\beta=1$, $\alpha=1$, $c_1=c_2=0$, $R\to R/2$ gives the fermionic contribution in the text, equation (2.6). Note that for $c_i\neq 0,1/2$ the sums in $\mathcal{H}$, $\mathcal{F}$ over positive integers cannot be written in terms of sums over the whole set $\mathbb{Z}$, as done in the text for their counterparts in equations (2.5), (2.6) with $c_i=0,1/2$. In such case the results in appendix B (with appendix A) which evaluate the corresponding sums over the whole set $\mathbb{Z}$ cannot be used anymore and one has to apply instead the results of this section.

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